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# Analysis of teachers' practices: The case of fraction teaching at the end of primary school in France

Cécile Allard

University of Paris VII, LDAR, Laboratoire André Revuz, Paris, France, [cecile.allardb@free.fr](mailto:cecile.allardb@free.fr)

*My study deals with Institutionalization Process in French primary school. Institutionalization Process (IP) is defined in the Theory of Didactical Situations (TSD) as a process enabling to decontextualize and depersonalize knowledge. I focused both on what decontextualizing and depersonalizing imply in teachers' actions and on knowledge exposure, proposed by teachers. I particularly focused on fraction teaching at the end of primary school. My methodology allowed me to collect everything that is said diffused about fractions.*

**Keywords:** Institutionalization, practices, fractions, decontextualization, depersonalization.

## INTRODUCTION

In TSD (Brousseau, 1998) also explains that the aim of TSD was the production of mathematical situations because there was a lack of such situations in teaching, so questions about institutionalization (knowledge exposure) arose much later. More recent works question this process and show in what manner it is important for teachers to take over didactical memory of the class (Brousseau & Centeno, 1991). Butlen (2004), Butlen, Peltier-Barbier and Pezard (2004) show that a tension exists between devolution and institutionalization for the benefit of devolution. Coulange (2012) obtains similar results to those of Butlen and colleagues (2004) and of Margolinas and colleagues (2002). Coulange talks about deletion of the formulation and knowledge exposure for the benefit of practices that puts ahead a “permanent shoring”. Observations and results of these authors lead us to question how expert school teachers (Tochon, 1993) deal with knowledge exposure. What are their constraints (Robert, 2001), which regularity and variability (Robert & Rogalski, 2002) will it be possible to

determine when they institutionalize (for their meaning as for our)?

## QUESTIONS AND OBJECT OF STUDY

In TSD, institutionalizing is defined as the action of depersonalizing and decontextualizing knowledge that arise following a situation of action. Pupils face a situation they need to solve by mobilizing knowledge and skills. The results of this situation lead them to build new knowledge. Brousseau (1998) thus shows one of the first institutionalization paradoxes: how to convince pupils that they have just learned something new despite the fact they were not able to solve the given problem. The teacher's main role here is to demonstrate and focus on this new knowledge. For this he has, on the one hand, to promote discussions between pupils in order for them to realize the great variety of their strategies involved to solve problems. On the other hand, they need teacher's support to agree due to their lack of a common language. In addition, the teacher has to show and name the new knowledge. These moments when the teacher names and shows the knowledge engaged will be called knowledge exposure.

Several recent French works (e.g., Butlen, Pezard, & Masselot, 2011; Coulange, 2012; Margolinas & Lappara, 2008) have shown difficulties met by junior faculty to deal with these knowledge exposures. Butlen and colleagues (ibid) show that teachers know how to devolve situations but meet strong difficulties to institutionalize knowledge. They then talk about tension between devolution and institutionalization. These difficulties to say and show involved knowledge at school, partly explain educational inequalities in France (some parents can help their children, some others not) (Rochex & Crinon, 2011).

A first study (Allard, 2009) has shown that written knowledge exposures, were few in number at the end of primary school (around twenty short texts for 36 class weeks for an average of 5 hours mathematics lesson per week). This small number of written traces let us assume the hypothesis that knowledge exposures are made orally most of the time. For my PhD I have tried to check this hypothesis.

These knowledge exposures can take place at the end of a lesson, at the beginning of another (reminder-phase), during the lesson, or during the correction of exercises. These knowledge exposures are quite fuzzy and I had to design a specific methodology to study them. We will describe it in a next part.

Although recent works are generally focused on junior faculty, we have decided to deal with skilled teachers. These teachers passed a specific exam (CAFIPEMF) to validate their expertise. These expert teachers are called PEMF (for "Professeur des Ecoles Maître Formateur"). The exam consists in three parts. The first part consists in presenting two lessons in two different disciplines in front of a five-people jury (inspectors, PEMFs, academic advisors, university professors). The second part takes place in the classroom of a pre-service teacher whose practice has to be analyzed by the candidate to CAFIPEMF. The final event consists in making a short presentation about his/her professional dissertation. These PEMFS are teachers that receive and train teacher trainees. In this paper, I will approach the following research question: How do these skilled teachers deal with knowledge exposure?

## METHODOLOGY

My methodology is comparative and qualitative. Comparisons are possible between several PEMFS but also, over several years for the same PEMF. Depending on comparisons, we had to use different methodologies. I followed five PEMFS over a year, including two for three years. In order to make comparisons easier, I set down some particular variables such as the use of a common manual and keeping to one mathematical problem: fractions. To make comparisons easier I filmed the teachers during all their lessons on fractions. In this paper, I will focus on the comparison of teaching practices of the same teacher for two years. I will therefore be able to talk about regularities and variabilities of these practices.

For this teacher, teaching fractions represents seven sessions of 50 minutes each for the entire year. I have transcribed all the lessons I filmed. These videos enabled me to retrieve a large variety of data. I mainly focus on the words expressed by the teacher depending on pupils' activities. I will particularly focus on the fidelity of the field of mathematics during three moments in the classroom: explanations given to pupils, before and after a research phase, and during knowledge exposure. These oral knowledge exposures are part of teacher-pupil dialogues. Through the analysis of these fragments of sentences, I will be able to put them together to recompose a text of knowledge as was shown to pupils. These knowledge exposures are studied according to their decontextualization level (Pezard & Butlen, 2003). I also kept the exercise books in which exercises were given and done and also the other exercise books that contained mathematical texts to be learned at home by heart. This data collection enables me to determine what kind of tasks pupils have done and what they have to learn at home. In France, it is forbidden to give written homework. Despite this prohibition that has been in effect since 1956, parents in certain social environments claim homework for their children. The only type of authorized homework, however, is reading and learning mathematical rules by heart.

These different data enable us to fill in the various practices components of PEMFs and to determine variabilities and regularities (Robert & Rogalski, 2002). I thus adopt the methodology described by Robert and Rogalski (*ibid*). These authors break down practice into 5 components (personal, social, institutional, cognitive and mediative) then recompose them. The personal component is only accessible by interviewing the teacher. It gives information on the relationship that the teacher has with mathematical knowledge, and choices made to help him/her carry out in comfortable manner his/her classroom teaching. The social component gives information on the teacher's working place, his/her colleagues and the social environment of his/her pupils (both in disadvantaged and advantaged areas). The official curriculum, the mandatory hourly amount of math lessons, the use (or lack of use) of certain handbooks and the relationship with inspection and inspectors, all give information on the institutional component. The cognitive component corresponds to teacher choices about content, tasks, organization and forecasts on how to manage. The mediative component is particularly relevant

since it deals with improvisations, speeches, pupils' participation, devolution of instruction and knowledge exposures.

Thus, in order to describe practices and inform mediative and cognitive components, I focused on the choice of handbooks, the mathematical problems, on exercises books and on the teacher's speech. I then had the necessary's data to know how things are said and what is transmitted to pupils.

## CASE STUDY

I chose to study the particular case of teaching fractions because it is a new notion that is introduced at the end of primary school. In French primary school, fractions are only studied under the subconstruct, called part-whole (Kieren, 1983). Introducing fractions is backed up by materials such as paper strips and circular areas.

I will name "Solene" the teacher I followed from September 2011 to June 2014. I previously followed this teacher from 2008 to 2011 but with a different focus. During these five years, I have collected enough data to describe her teaching practice carefully. 2011 was the first year when she had to deal with a double level class (CM1 and CM2: from 9 to 10 year old pupils). That year she proposed only three sessions on fractions. These sessions consisted in coding shapes. Shapes were subdivided into several equal parts. Fractions can be described by a numerator (colored area, part of a whole) over a denominator (all the parts, the whole). Solene proposed little written knowledge exposure. In 2012, she did not teach fractions to her group of CM2, but her colleague carried out this teaching.

We are now going to describe the five components found in Solene's particular case.

### Personal and social components

Solene have a good personal relationship with mathematics. She declares that she loves mathematics and considers that she has a solid knowledge in this field. She has a Master degree in the field of Developmental Psychology. She is confident enough in her choices and her professional skills to use the resource designed by the ERMEL team (a group of PEMFs, didacticians and mathematics teachers). She has been using this resource since 2008 even though this handbook is considered to be difficult.

Solene has been a teacher for 15 years, 10 of which she served as a replacement teacher. In 2008, she took the responsibility of a class in a rural school and she still teaches there today. She passed the specific exam to become a PEMF (CAFIPEMF), in 2013. She collaborates with a colleague (also a PEMF). The school team is stable. Pupils who attend the school have a good general scholastic level (above average for national evaluations), and none of the parents are unemployed.

I can conclude that Solene has a good relationship with mathematics and works with a good team in a pleasant, working environment.

### Institutional component

To describe this component, it is necessary to study the official curriculum proposed by education department in 2008. In this curriculum, teaching of fractions is spread over the last two years of primary school. The proposed progression is dedicated to decimal building "from fractions to decimal fractions". It relies, on Perrin-Glorian and Douady's (1986) work without explicitly naming them. The progression is built over several steps: learn, name and write fractions of the unit, know how to break up fractions into the sum of a whole number and a fraction of unit.

After this introductory work is carried out, teaching of fractions will begin. Decimal writing or "with dot" are introduced as a different way of writing, after the one of decimal fractions, to name decimals.

Arditi (2011, p. 95), in his PhD on the use of a written resource by didacticians, explains that, "the study of fractions is only a prerequisite to install decimals". Note that the French program adds an indication on using fractions in a particular context which is the coding of size measures. The official instructions have not changed over the three years of my study even though, in France, they are often updated. The last versions occurred in 1995, 2002, 2008 and will change again in 2015.

Solene uses the ERMEL handbook, which is recommended during teacher training. This resource is appreciated by the inspectors. In France, inspectors come to observe teachers in their classrooms. They check that the curriculum is followed conforming to official instructions and, finally, they evaluate teachers. This evaluation is important for career development.

### Cognitive component

Solène spent seven sessions devoted to the teaching of fractions (including evaluation) and then continued with decimal fractions. Solene introduced fractions using paper strips. Using these strips, pupils draw and measure segments whose lengths are expressed under the form of fractions smaller than a unit or as the sum of a whole and a unit fraction. To understand what is proposed to pupils, it is important to describe the activity presented by the teacher. I noted that the cognitive path was identical in 2011 and 2013, because same activities and same instructions were given to pupils. The global project remained stable.

The first situation proposed to pupils by ERMEL aims to:

- Set in mind the first meaning of simple fractions:  $1/2$ ,  $1/4$ ,  $3/4$ ,  $1/8$ ,  $3/8$ .
- Know and use the relationships between fractions, express them in multiplicative and additive writings.

Ermel offers two activities. In the first activity, the pupils are asked to fold a paper strip in order to cut out three quarters of the strip. Then an additional instruction is given: from a strip measuring three-quarters of a unit strip, pupils have to restore the unit strip. Below is a brief description of this activity.

The unit strip is named A. Pupils have two strips of the same width (B and C strips).

- B strip longer than the A strip
- C strip is three quarters the length of A strip.

The “A-strip” is shown on the board. The “C-strip” has two folds to visualize each quarter. Pupils are now given the task on the strip B to duplicate four times a quarter, two times a half or even duplicate

the “C-strip”, and then add one quarter. “C-strip”: can be read in two ways  $\frac{3}{4} + \frac{1}{4} + \frac{1}{4}$  or  $\frac{1}{2} + \frac{1}{4}$ .

Solene has proposed this same activity for three years, and her instructions were always the same. Despite the similar teaching pattern, variations have also been noted. The first session is organized in the same way, and pupils carry out mental arithmetic on doubles and halves. Solene then goes on to the second step of the session by presenting the paper strips. Even though questions and global project stay identical, significant differences arise according to the duration of the sessions.

The first session lasted 48 minutes in 2011 and 90 minutes in 2013. In each year, the session is divided into three main parts (Table 1).

The duration of these three parts is well balanced, but, in 2013, the entire duration of the session was doubled. For all the other sessions I observed, the time for each session was also doubled. A notable difference can be explained by taking into account what the pupils have to say. In 2013, Solene clearly understood this situation and was ready to listen to her pupils.

### Mediative component

To describe this component, I will rely on the transcription of what the teacher says about mathematics. I will then propose possible explanations of why the duration is doubled for the same session.

In 2011, Solene proposed ten calculations on halves and doubles. Solene spent a very short time on observing pupils procedures. She only listened to pupils who found the correct answer, and she concluded by giving the following rule: “cutting a number or figure is two times less, cutting a number or a figure by four is four times less”.

The second part of the session is dedicated to the manipulation and folding of strips and to understand

	2011	2013
Part 1: thoughtful calculation : calculate doubles and halves	21 minutes	30 minutes
Part 2: fold a strip and cut three quarters. (see instructions for session A in Table 2)	10 minutes	30 minutes
Part 3: make a one unit strip from the three quarters of this unit strip. (see instructions for session A in Table 2)	17 minutes	30 minutes

Table 1



what a fraction of the unit (context part/whole) can represent.

In 2011, it was the first time the teacher introduced this concept, using the support of paper strips. The teacher exposed knowledge at two moments. The first moment occurred just after bringing the procedures together. For example, she said that “four times one quarter is equal to a unity” without writing the number one.

- 46 Teacher: So initially, you have a half plus a half equals two halves (this written on the board as  $\frac{1}{2} + \frac{1}{2} = 2/2$ ), you have cut each half in half and four quarters ( $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ ) is equal to a unit. Is that okay?

At the time the aim of the lesson was given, Solene showed what she wanted to teach but keeping within pupils' vocabulary. She oriented her explanations towards resolution strategies. Knowledge exposures were targeted to the exposure of resolution strategies.

- 82 Teacher: I have a question ... the two strategies that have been used to do the work, is to add a quarter. Second strategy ... to use a half, to reproduce a half to obtain my strip. That's it! Perfect.

In 2013, the global scenario (Butlen, 2007) was the same with a view to the written preparations of the teacher. It is at the micro level that differences arise. There was no conclusion at the end of mental arithmetic. The teacher asked pupils to expose their strategies “a quarter is half of a half”, and explained that it was more difficult to find the half of 24 than of 38. In 2013, the teacher let pupils elaborate more about their procedures. I can note that half is defined as the action of “cutting by two” and “it's two times less”, whereas a quarter is defined as “the half of a half”. Same expressions are used with reference to the school equipment: “cut in two”. Two pupils did not manage to carry out these calculations correctly.

I can only hypothesize that pupils' knowledge is acquired by action. When pupils expose their strategies, they used their current vocabulary language instead of mathematical terms. They refer to actions as “cutting by two”. The teacher also uses pupils' vocabulary, which may not help the pupils learn and understand mathematical terms.

The teacher questions numerous pupils and spends a long time explaining their strategies again without proposing other reformulations. The teacher uses pupils' vocabulary and does not propose other more “mathematical” vocabulary.

Most of the pupils manage to carry out the task, and it seems satisfying enough to go ahead with the rest of the session. The following session consists in folding sheets of paper to represent what one half is.

I therefore think that this phase of mental arithmetic was a means for the teacher to recall the knowledge of her pupils on what a quarter and a half are. The teacher reminds pupils of knowledge in a numerical context. I note that taking  $\frac{1}{2}$  of a number (operator point of view) is not the same point of view as the fraction which is as considered a part of the whole. Whether or not pupils are able to take this into consideration without the help of the teacher is hard to tell.

During this second step, Solene finally said and wrote that four quarters are “four times a quarter, but also three times a quarter plus one quarter”. A pupil proposed “three quarters plus two quarters minus one quarter”. This indicates that pupils are capable of producing equalities on fractions by the use of strips. They therefore produced more varied different equalities than in 2011, but all references to the unit disappeared.

At the end of the session, the teacher asked what they have learned today. She did not do this in 2011. Two pupils were capable of answering. They said that they had “learned quarters and halves” or “strips”. The teacher concluded her lesson and reminded her pupils of the successive interventions. She proposed what seemed to be a synthesis of what was said: “These are fractions; we learned to fold strips to get quarters, three quarters in a strip plus several fractions enable to reconstruct a whole strip, OK?” The synthesis underscores actions on school equipment and vocabulary used.

These knowledge exposures are not prepared in advance. The knowledge exposure moment, is planned, but the content isn't. The knowledge exposure is produced in action after a discussion with the class. They re-use certain pupils' terms, excluding others. Fraction is defined by reference to an action using

school equipment. Pupils are responsible for understanding that the whole strip is a reference to the unit.

632 Teacher: the halves ... we learned the strips? We have not learned strips. We learned to do what with these strips? Three halves ... no, we did not learn to do three halves, we did quarters. How do you call this? (Shows a fraction) What is called?

635 Pupil: fractions

636 Teacher: These are fractions, we learned to fold the strips to get quarters, three quarters in a band plus several fractions allow us to rebuild a whole strip. Ok?

During the following lesson, Solene distributed a text to be learned by heart at home. This text was the same in 2011 and in 2013. Whereas she spent 20 minutes to exploit this text in 2011, it took her more than an hour in 2013. Pupils were asked in 2013 to comment on every line of the text that was distributed. Once again, I noted this difference in duration and the attention given towards what the pupils had to say.

## CONCLUSION

For two years, I followed 5 teachers, but in this paper, I have only developed one case. The knowledge exposure does not develop mathematical exposure but develops methodological knowledge much more. These institutionalizations (knowledge exposure) remain strongly contextualized. The context does not remain entirely the same from one year to another and depends on interactions with pupils.

The study of the five components leads us to point out regularities and variabilities. For this teacher concerned, the teaching project is consistent concerning official programs and didacticians' recommendations (context of lengths). Apparently, the teaching project remains the same, although differences appear. I question these differences. The variabilities I noticed appear at the level of the mediative and cognitive components and more precisely in the content of knowledge exposure moments. What the teacher says orally seems randomly improvised and depends on discussions with the pupils. The oral content of the lesson leans on the pupils' vocabulary and moves away from mathematical vocabulary.

The game of questions/answers, difficulties met to name mathematical objects, to express the target taught, lead me to introduce the notion of negotiated institutionalization.

Expecting the pupils to be able to say what they have learned at the end of a session seems to be quite ambitious. Taking into account what the pupils declare they understood, by using imprecise vocabulary enables them to formulate and share their newly acquired knowledge between their peers. It can be questioned, however, if it is reasonable to propose incorrect and unclear mathematical definitions. This leads me to ask myself new questions oriented towards the practices of training teachers and more precisely on their professional gestures. What mathematical and didactical knowledge should a teacher acquire in order to achieve the transition from contextualized institutionalization to de-contextualized institutionalization?

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